

For the past few weeks, we have been focusing on Divisibility and Remainders. There are some more 'types' of remainder questions. Let's take them one by one so that by the end of it all, you are an expert in everything related to remainders. In this post I will start with a question similar to what you might find in the *Official Guide for GMAT Review*.

Question: If  $a$  and  $b$  are positive integers such that  $a/b = 97.16$ , which of the following cannot be the remainder when  $a$  is divided by  $b$ ?

- (A) 4
- (B) 12
- (C) 22
- (D) 28
- (E) 96

Some of my most brilliant students have asked me something similar to this: "When I divide 11 by 4, I get 2.75. What do you mean by, 'What is the remainder?' Where is it?" So if it is bothering you as well, don't worry; I will address this issue first.

Say, I tell you the following: Divide 11 by 4. What do you get?

You could answer me with one of the following:

Case 1: You could say, "I get 2.75"

Case 2: You could say, "I get 2 as the quotient and 3 as the remainder."

Either way, you are correct.  $11/4 = (2 \frac{3}{4})$

When you use the decimal form, you get a .75 which you add to 2 to give you 2.75. This .75 is nothing but the way you express the remainder 3. When you divide 11 by 4, 4 goes into 11 two times and then 3 is left over. When 4 goes into 3, you get 0.75 which is  $\frac{3}{4}$ . That is the reason why you can write  $11/4$  as  $(2 \frac{3}{4})$  in mixed fractions.

Do the following calculations and see what you get. Express the answer in both the forms – Decimal form and quotient-remainder form.

1. Divide 22 by 8.
2. Divide 55 by 20.
3. Divide 275 by 100.

Let's see what we get in each case.

1. Divide 22 by 8.

Case 1: In decimal form we get 2.75

Case 2: In quotient-remainder form we get 2 as quotient and 6 as remainder (Remember, the divisor is 8 here). We can say  $22/8 = 2 \frac{6}{8}$  in mixed fractions.

1. Divide 55 by 20.

Case 1: In decimal form we get 2.75

Case 2: In quotient-remainder form we get 2 as quotient and 15 as remainder (Remember, the divisor is 20 here). We can say  $55/20 = 2 \frac{15}{20}$  in mixed fractions.

1. Divide 275 by 100.

Case 1: In decimal form we get 2.75

Case 2: In quotient-remainder form we get 2 as quotient and 75 as remainder (Remember, the divisor is 100 here). We

can say  $275/100 = 2 (75/100)$  in mixed fractions.

Notice that the remainder is different in each case above. As the divisor changes, the remainder changes even though in decimal form the answer is the same. This is so because in each of the above cases,  $6/8 = 0.75$ ,  $15/20 = 0.75$  and  $75/100 = 0.75$ . Each of these fractions is equal to  $\frac{3}{4}$ . Now if I flip the question and say, "When I divide a by b, I get 2.75. Which of the following cannot be the remainder when a is divided by b: 2, 3, 6, 12, 15?"

We saw above that  $2.75 = 2 (75/100) = 2 \frac{3}{4}$  in the lowest form. So the remainder will be 3 if the divisor is 4. The remainder will be 6 if the divisor is 8. The remainder will be 15 if the divisor is 20. On the same lines, the remainder will be 12 if the divisor is 16 because  $12/16 = \frac{3}{4}$ . Can the remainder be 2?

$$3/4 = 2/?$$

We cannot have an integer value in place of the '?'. Hence we will never get 2 as the remainder. The remainder will always be a positive multiple of 3.

Let's go back to the original question now: If a and b are positive integers such that  $a/b = 97.16$ , which of the following cannot be the remainder when a is divided by b?

$$a/b = 97.16 = 97 (16/100) \text{ in mixed fraction} = 97 (4/25) \text{ in the lowest form.}$$

The remainder must be a positive multiple of 4. 22 is not a multiple of 4 hence you can never have 22 as the remainder

$$4/25 = 22/?$$

You cannot have an integer in place of the '?'

Hence answer is 22.

Let me end this post with a question for you: If a and b are positive integers such that  $a/b = 82.024$ , which of the following can be the value of b?

- (A) 100
- (B) 150
- (C) 200
- (D) 250
- (E) 550

Hopefully, you will arrive at the answer in a few seconds!